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Modeling the Dynamic Behavior of Concrete

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Abstract

A literature search was undertaken to assess the state-of-the art in modeling the response of concrete structures to short-duration loading. The literature on concrete is vast, both on its constitutive modeling and the response of structures or structural components to various types of loading. Many structural geometries have been considered and a number of scholarly journals and a countable infinity of books, technical manuals and reports exist. Almost exclusively, these deal with the static behavior of concrete, that is, situations where the loading rate ranges from 0.0001/s to 0.01/s. Very little is available for dynamic loading, i.e., strain rates > 0.01/s. The main problem areas of interest are impulsive loading of structures and structural elements [e.g. blast, vibration and earthquake loads] and impact loading by kinetic energy and chemical energy projectiles [e.g. fragment and penetrator impacts resulting in penetration, perforation and extensive cracking, demolition of concrete structures by cutting charges, penetration of concrete runways and bunkers by bombs and the damage resulting from detonation of the explosive]. These latter phenomena tend to occur at strain rates exceeding 100/s. The intermediate regime between these two, referred to often as soft loading and occurring at strain rates of 0.01/s - 100/s, involves impacts of both rigid and deformable strikers against concrete structures which are not perforated but can be severely damaged through cracking, excessive bending or failure of a critical component. Design work is based mainly on empirical equations or simple analytical models which call for minimal input on concrete behavior, typically its tensile or compressive strength. Advanced models exist and calculation with structural dynamics [e.g. ABAQUS] and wave propagation [e.g. DYNA] computer codes have been done. These suffer from a number of problems:

- a lack of fundamental physics in the constitutive model for concrete. Such codes were originally designed for use with metals and metallic structures. The concrete models therein are simple extensions of metal physics models with additional parameters to limit tensile loads and other well-known aspects of concrete response.

- a lack of data on the constitutive behavior of concrete at high strain rates. Some data exists from split-Hopkinson bar experiments and plate impact tests, but not enough to discriminate between the various constitutive models for concrete and concrete structures that have been proposed.

- a lack of controlled experiments to allow validation of computer predictions at both global and local levels.

This report summarizes the information currently available in the literature and recommends ways to improve existing models for the behavior of concrete and concrete structures under short-duration loading.

Caveat Emptor

This report is a review of the relevant literature on the dynamic behavior of concrete and concrete structures. It is not a comprehensive review - it covers only material the author considered to be relevant. The determination of what was, and what was not, relevant, was made by the author relying on a rigorous education in physics, applied science and engineering mechanics at three major universities, considerable personal experience in the field of structural dynamics and impact mechanics over a span of more than thirty years and consultation with colleagues with established reputations in the field on various aspects of the topic. These latter will remain unnamed, as is the custom for unindicted co-conspirators. As, therefore, this report represents the *opinion* of its author supported by what he considers to be relevant references on the subject matter covered, any errors or sins of commission are strictly those of the author for which he is solely responsible. Should some portion of this review be deemed to have merit, it is highly probable that this is due to the excellent advise received by the author from his confidants and reflects credit on him only insofar as he chooses his friends and associates judiciously and exploits them liberally.

Introduction

Concrete is a universal construction material. Many structures originally made of steel are now built with concrete. Not counting buildings, roadways, runways, bridges and other everyday structures familiar to most people, concrete has been used to build barges and vessels, geothermal power reactors, power and diversion tunnels for hydro-electric schemes, prestressed concrete nuclear reactor vessels, containment vessels, nuclear shelters, silos and bunkers, offshore structures and liquefied natural gas tanks. This represents but a small sampling of the use of concrete. There is also the potential for the use of concrete in totally novel applications such as concrete lunar structures [King (1993); Miyamoto and King (1994)].

As the scope of application increases, the surrounding environment in which these structures must operate can become severe, increasing safety requirements in design. Dancygier, Yankelevsky and Baum (1999), for example, present an experimental study on the impact response of protective reinforced concrete structural elements that are coated with plaster on their interior faces. Even when a structural element's perforation is prevented, impact conditions may initiate rear face scabbing due to high-intensity reflected stress waves at the element's rear face. The fragments that result from this scabbing failure can travel at speeds fast enough to injure civilians in dwellings and civil defense shelters.

Impact or impulsive loading resulting from collisions is a particular case familiar to the general public. Nuclear reactor containment structures must be designed to prevent leakage of radioactive material into the atmosphere from impact by aircraft, automobiles and assorted tornado-borne debris. Various methods of energy absorption must be incorporated into concrete structures located in portions of the planet subjected to frequent earthquakes. Impact of ships on bridge girders and piers and offshore marine platforms is another well-known phenomenon often reported in the news.

A particular difficulty in dealing with these problems is the lack of a rational and globally acceptable design method or procedure for structures that are susceptible to short-duration loading. This is true for metals and metallic structures. It holds particularly for concrete. The ultimate behavior of concrete structures is very complex due to material nonlinearity in concrete cracking, interaction between concrete and any reinforcement, shear transfer at cracked sections and other considerations. Not the least is that in dynamic situations wave propagation effects must be considered together with failure mechanisms (scabbing, spalling) not observed under static loading. Johnson (1972) has observed that in ordinary demolition of buildings with prestressed concrete, the cutting of a reinforcing bar which is in compression can result in waves traveling in the bar. On contact with a free surface, these waves change character from compression to tension and tension waves are generated in the neighboring concrete. Since concrete has little tensile strength to speak of, showers of concrete particles are emitted from the area, with some particles traveling at velocities approaching 100 m/s. At these velocities, the concrete particles can be lethal to people in or near the construction site.

The loading conditions for concrete structures range from the ***static*** (the loads are independent of time), ***dynamic*** (time-dependent loading), ***gradually applied*** (the magnitude of the loading changes on a time scale that is large compared to the natural period of the system), ***harmonic*** (the loading changes with some regular manner with time) and ***transient*** (loads are applied and changed in a non-harmonic way, usually exciting the high-frequency response of the structure). The ***transient*** loads can be further classified depending on the total duration of the load, pulse shape and the rise time to peak stress or pressure.

a. ***impact loading***. Impact involves contact between two bodies. The peak magnitude of the load and its variation with time depends on the masses involved. In brief, there is an interaction between the mass of the striker and the mass of the target which influences both the impact load and the deformation response of the target and striker.

b. ***impulsive loading***. Loading of this type does not involve contact between two masses. Examples are air blast, wind gusts, shocks from explosions. The magnitude and variation with time of the load may be comparable to those from impact loading, especially for explosive shocks. In general, though, peak load duration and response times are orders of magnitude longer than for impact situations. They are also largely independent of the mass of the loaded structure and its manner of deformation.

c. ***earthquake loading***. The mass of the structure and its deformation are important. The prime load is a sequence of rapidly imposed displacements on the base of the structure.

d. ***other dynamic loads***. Examples in this category include unbalanced oscillating machinery, wheeled traffic, sudden removal of load-bearing elements of a structure. Many more examples can be found in Pilkey and Pilkey (1995).

The various methods devised to analyze concrete structure under these loading conditions include empirical approaches (by far the dominant method for structural design), analytical modeling and numerical methods. These latter include finite differences, finite elements and discrete element approaches. Each will be discussed in turn. By way of background reading, Appendix A contains information on numerical modeling of nonmetallic materials.

Computing the Structural Response of Concrete Structures to Short-Duration Loading

The approaches to calculating the behavior of materials and structures can be grouped into three categories: empirical, analytical and numerical [Nicholas (1990)]. A similar grouping is possible for concrete. Much of the information on concrete dynamics is grouped in a few sources. This is most fortunate for researchers in this field. Beyond these, information on the dynamic behavior of concrete is scattered over a diverse literature and company reports.

The most readily accesible sources of information are:

a. the International Conferences on Structures Under Shock and Impact (SUSI). The conferences are organized by the Wessex Institute of Technology. Proceedings are published by Compuational Mechanics Publications in Southampton, UK and Billerica, MA in the United States. The latest in the series, SUSI VI, will be held in the summer of 2000.

b. the International Symposium on Interaction of the Effects of Munitions with Structures deals extensively with the behavior of concrete under blast and fragment loading. The emphasis currently is on the protection of existing civilian structures against terrorist attack. The symposium draws participants from around the world and covers all aspects of the dynamic behavior of concrete structures. Proceedings are published but there is no systematic method of distribution.

c. the Symposium on Structures Under Extreme Loading Conditions, part of the annual Pressure Vessels and Piping Conference of the American Society of Mechanical Engineers, usually has a large number of papers dealing with concrete under short-duration loading. Papers are published in a separate volume.

d. the annual Bombs and Warheads conference spronsored by the American Defense Preparedness Association (ADPA) and sundry U. S. Defense Department agencies emphasizes the response of military structures of concrete, such as bunkers and runways, to impact and explosive loading.

e. the International Symposia on Ballistics, also co-sponsored by ADPA, is also a good source of papers on concrete behavior presented by an international audience.

f. conferences are readily held in Japan on the behavior of concrete structures subjected to static and dynamic loading. Among these are:

the International Symposia on Impact Engineering
the International Symposia on Impact Problems Related with Falling Rocks

g, the SMiRT conferences (Structural Mechanics in Reactor Technology) contain a wealth of information on the uses of concrete for nuclear reactor applications. One section of this very extensive conference is devoted to problems in dynamics. Extensive proceedings are published.

empirical approaches

As in other impact situations, this is the predominant approach due to various deficiencies in analytical and computational techniques. All building standards are based on empirical formulas. Much of the U. S. experimental work on blast and impact is summarized in two U. S. Army technical manuals:

TM5-1300, Structures to Resist the Effects of Accidental Explosions, Department of the Army, 1969

TM5-855-1, Fundamentals of Protective Design for Conventional Weapons, Department of the Army, Corps of Engineers, 1984

Computer software has been developed based on the information in these documents that allows rapid, parametric studies of various loading scenarios.A short course developed by Prof. Ted Krauthammer at Penn State University called *Modern Protective Structures* makes extensive use of the material in these documents. Many examples of the use of empirical methods to design concrete structures are to be found in the proceedings of the various conferences mentioned above.

analytical methods

Since analysis techniques for concrete have been shown to be very sensitive to the nature of the loading function, much of the effort here is to determine the loading as accurately as possible using experimental data [von Riesemann et al (1989)], rigid body methods [Sonoda, Kobayashi and Matsuura (1994)] and other ad hoc techniques which may be coupled with finite elements or finite differences [e.g. Barondes et al (1997)]. This

is followed by an assessment, ranging from simple judgement that a critical parameter has or will be exceeded during the event time to detailed structural calculations of the damage done to the target. For this latter aspect, fracture mechanics techniques are gaining popularity, especially in Japan.

Consider, by way of example, the approach by Sonoda et al. Their interest is in designing reinforced concrete rock sheds. The rock shed is a portal frame covered with a sand cushion. It is desired to determine the response of the structure under different loading conditions such as varying the rock mass, the drop height, angle of impact etc. These loads are considered in addition to dead loads and soil pressure.

Almost all structural design codes in Japan use an equivalent static load procedure. A static analysis is performed using a loading equivalent to the dynamic peak load expected. Since it ignores wave effects, this procedure leads to a very conservative design.

Sonoda et al use a rigid-body-spring mass system first proposed by Kawai (1980). The portal frame of the rock shed is divided into a number of rigid blocks along the frame axis Figure 1, and material constitutive relations for concrete and reinforcements are only applied to the springs distributed over the interfaces of the rigid blocks. Stress-strain relations in compression, shear and bending are used in the analysys and a fracture energy criterion is used to determine failure, Figure 2. The loading function is shown in Figure 3. Results for normal and oblique incidence are in Figure 4. As is commonly done, reinforcements are modeled as perfectly plastic bodies. The equations are straightforward and solutions to various impact scenarios are readily determined on small computers. A more elaborate model of this type is given by Miyamoto and King (1994). The main advantages of such procedures are tractable analyses (with the help of a personal computer) and relatively simple requirements for constitutive modeling. Thus, such models make good intermediate tools between purely empirical approaches and the more complex structural or wave propagation code solutions.

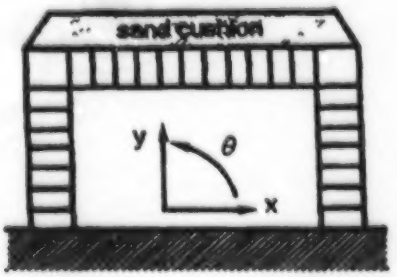


Figure 1. Discretization of Concrete Portal

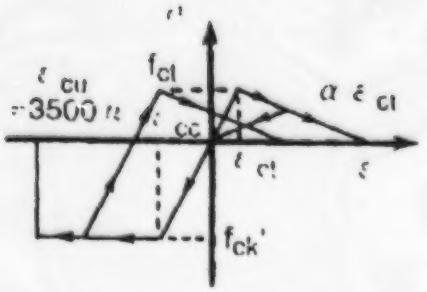


Fig 2 σ' – ε curve in concrete

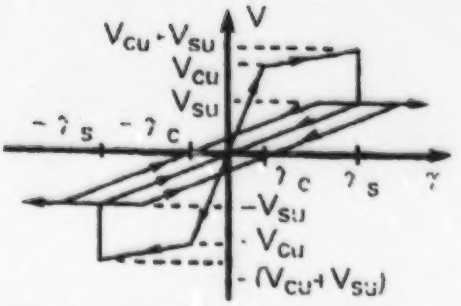
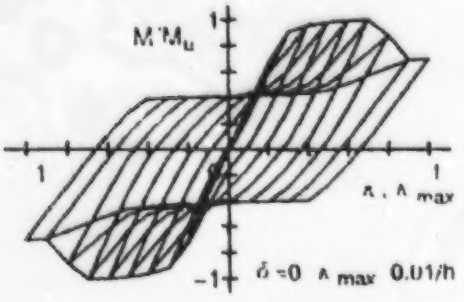
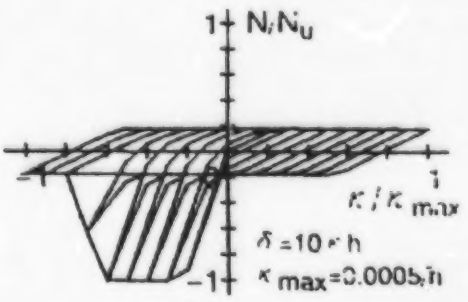


Fig.3 Shear force – strain curve



(a) M–κ relation



(b) N–κ relation

Fig.4 Hysteresis curve of RC section

Figure 2. Material Model

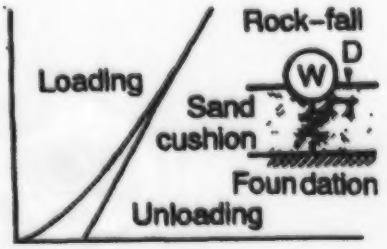


Figure 3. Contact Force Description

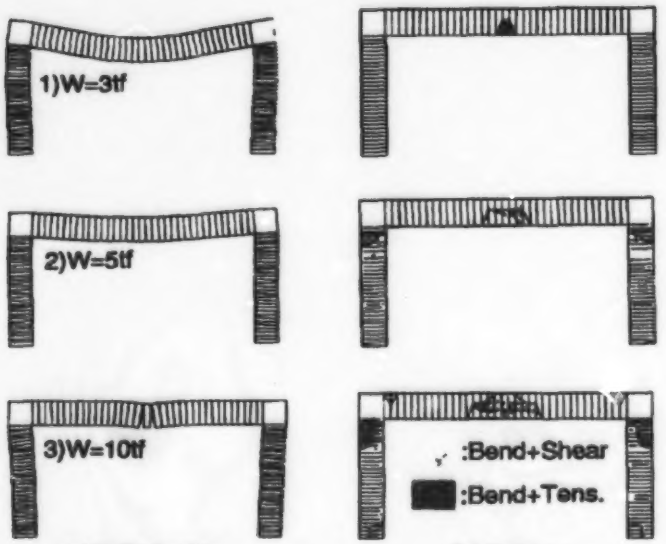


Fig.9 Deformation and material states (vertical fall : H=30m, T=0.096sec)

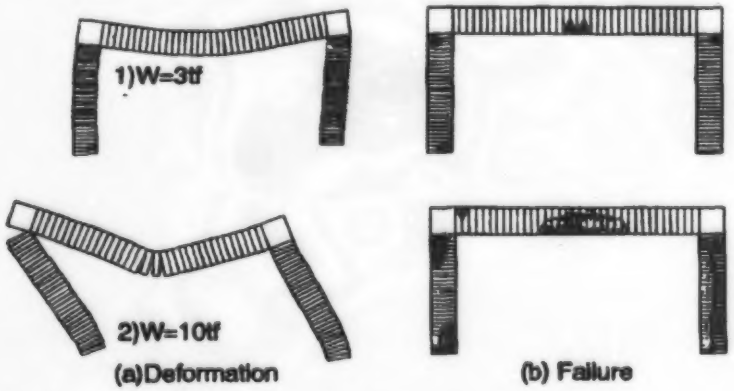


Fig.10 Deformation and material states
(inclined fall : H=50m, T=0.096sec)

Figure 4. Deformation and Material Sates for Normal and Oblique Impact

numerical methods

A theme repeated over and over again in many papers which make use of such codes as DYNA, EPIC and many others is that the codes cannot be used to predict the

behavior of concrete structures under short-duration loading. Rather, once calibrated to a particular data set, they can be used to infer additional detail on structural response that cannot be obtained from experiment. In this, codes with models for concrete are analogous to currently used reactive hydrodynamic codes for explosives. These also must be calibrated to experiments. They cannot be used to extrapolate to other conditions save for those for which they were calibrated, but can provide more information, especially about local behavior, than can be obtained from experiment.

Numerical simulations of concrete behavior at high strain rates is at about the state today where similar calculations with metals were 30-40 years ago, to wit:

- there are no definitive constitutive models
- there is a severe lack of high strain rate data for concrete
- there is a lack of controlled experiments which could be used to validate both local and global predictions of codes

In the early 1960's, when HEMP, OIL , HELP and TOODY were the primary wave propagation codes, calculations of impact and explosive loading events which occurred at strain rates of 100000/s or higher were done with data obtained from tests conducted at strain rates of 0.0001/s. Split-Hopkinson bars were a laboratory curiosity. High rate data was virtually non-existent. Hence, computational results hardly ever agreed with experiments. There then followed knob-twistings of all persuations. One analyst would change a material's shear modulus by a factor of four. Another would double the yield strength. A third would play with the bulk modulus. Another round of calculations ensued and sometimes, purely by chance, one of these would agree with experiment. Since instrumentation was not available then to measure wave motion or other constitutive parameters, comparison was of necessity made with global variables such as displacement or velocity. These, as time showed, were necessary but not sufficient discriminants of constitutive behavior. Many years went by before a data base of high rate properties was built up and a body of instrumented impact experimental data came into existence so that code calculations could be validated and performed with a good deal of confidence.

Much the same state exists today with numerical modeling of concrete. There are no definitive constitutive models. A number of viscoplastic models exist. These have been tested against experiments and often seem to produce reasonable results. Whether this indicates that there is indeed plastic behavior on concrete of whether the viscoplastic models simply lend themselves to easy calibration and echo back the desired results still must be determined. Wave propagation experiments for concrete exist. See the reviews by Ross (1997) , Tedesco and Ross (1997) , Malvar and Ross (1998) as well as Ross (1989; 1990), Bischoff and Perry (1991) for examples. Strain rate effects in steel reinforcing bars are reviewed by Malvar (1998). Kulkarni and Shah (1998) compare the response of beams at high and low strain rates. However, there is not enough of this and under a sufficiently wide range of impact and impulsive loading conditions to be able to unequivocally state the requirements for a constitutive model for concrete. Equation of state data for high pressure response is sparse at best [Gregson (1972), Grady (1996)]. There is some

question whether modeling reinforcements as elements consisting of layers of concrete and steel is a sufficiently general model, especially for kinetic energy impacts, even though it has worked well for computing gross deflections in some blast loading situations. Finally, if high rate data were available, there would still be a need for highly instrumented controlled experiments at velocities ranging from those where the penetrator remains undeformed throughout the event time through hypervelocty impact where catastrophic failure of colliding material occurs. Even if all this were available, there would still remain the question of the failure of concrete under high rate and high pressure loading.

Krauthammer (1994) summarizes the presentations, discussions, conclusions and recommendations of a 1993 workshop on structural concrete slabs under impulsive loading. Regarding numerical analyses, the report notes that classical (analytical) methods are limited to simple geometries. Computer codes have evolved into useful tools and this evolution is expected to continue. The most important current needs are :

- reliable constitutive models for a wide range of materials. To this we may add, based on our experience with metals, that the free parameters in the constitutive models must be obtained from experiments performed at strain rates appropriate to the problems being addressed.

- physics-based user-friendly approximate computational tools that have been validated and verified against reliable experimental data. This experimental data will presumably come from controlled, well-instrumented experiments that will yield not only global but local data. Again, judging by our experience with metals, such experiments will be difficult to perform and very costly. However, they are indispensable for validating computational and constitutive models. The good news is that not too many will be required.

Computers have gotten faster since Krauthammer's paper was written and more and more simulations of the impact or impulsive loading of concrete have appeared in the literature but the basic physics remains as Krauthammer described it. Many calculations, and some constitutive models, employ static yield data for concrete for want of dynamic data. Many ad hoc models exist. The more popular models working their way into codes assume concrete to be a viscoelastic or viscoplastic material. Some 2 - 5 parameters for the constitutive model must be determined from independent experiments. data for this purpose is scarce, so some of the total parameters needed constitute a best guess on the part of the user. The most popular forms are a crushable material model, following the p-alpha model first described by Herrmann (1969), and a CAP model. See Appendix A for a brief summary of both. The calibration of these models is a nontrivial exercise. As an example, see the procedure for determining the relevant parameters for a CAP model as described by Gupta and Seaman (1979).

Before summarizing the state-of-the-art in this area, it would be beneficial to look at the specifics of some of the more commonly used computational models.

The Osborne Model

This was one of the first models introduced into wave propagation codes. It now exists as equation of state #4 in the HULL code. A brief description, courtesy of John Osborne of Primex Technologies OTI Group, is in Appendix B. The HULL is built up from data reported by Gregson (1972), McHenry and Karni (1958) and Chinn and Zimmerman (1968). Gregson provides Hugoniot stress vs particle velocity data up to values of 500 kb for a small aggregate concrete. McHenry and Karni provide failure data for concretes varying in strength from 3000 - 6500 psi (207×10^6 - 450×10^6 dynes/cm^2). Chinn and Zimmerman provide yield strength vs confining pressure data for concretes varying in strength from 5200 - 8700 psi (360×10^6 - 600×10^6 dynes/cm^2). The Hull equation of state arbitrarily cuts off increases in yield strength at 30 times the unconfined compressive strength of concrete. The value of the cutoff was selected at the time because there was no available data beyond that point. Recent comparisons with plate impact calculations indicate a value of 50 gave a better fit to the experimental data. The Hugoniot form of the EOS is shown in Appendix B. It clearly falls into the category of Herrmann's crushable material model.

The EPIC Concrete Model

Over the years EPIC has evolved into a complex code which is now capable of straightforward Lagrange calculations of dynamic events as well as SPH calculations which may be run separately or coupled with a Lagrangian grid. Many applications of the code in two- and three-dimensions to plane and reinforced concrete impact are given by Johnson (1997). The concrete strength, pressure and failure models are shown in Figures 5 and 6 below.

Concrete Model

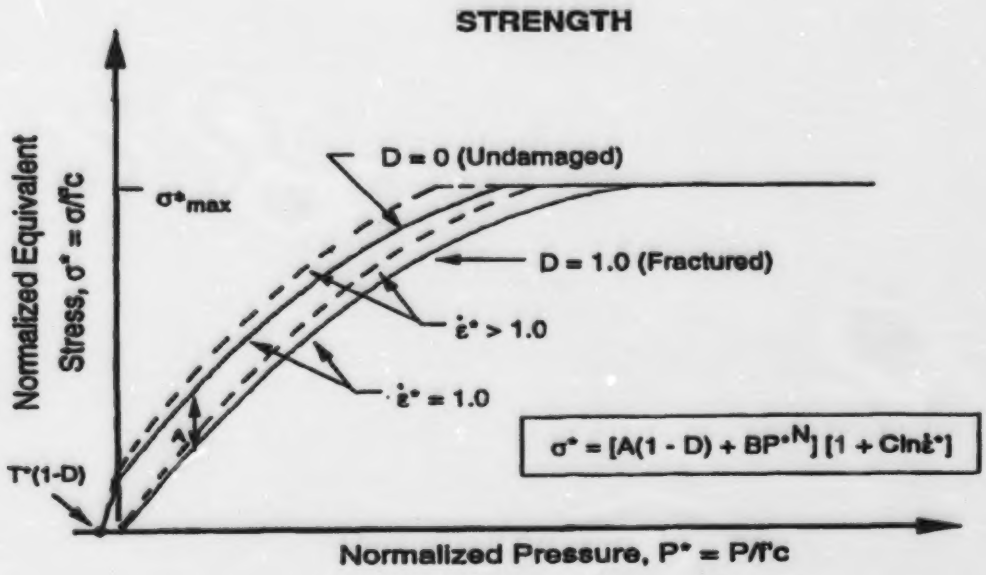

STRENGTH
Normalized Equivalent Stress, σ* = σ/f'c
σ*max
D = 0 (Undamaged)
D = 1.0 (Fractured)
ε̇* > 1.0
ε̇* = 1.0
σ* = [A(1 - D) + BP*^N] [1 + C ln ε̇*]
T*(1-D)
Normalized Pressure, P* = P/f'c

Figure 5. EPIC Model for Concrete Strength [Johnson (1997)]

Concrete Model

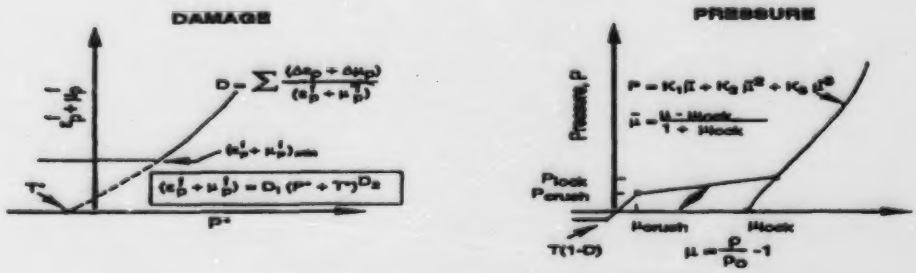


Figure 6. EPIC Representation of Damage and Pressure in Concrete [Johnson (1997)]

EPIC has been used extensively for a wide variety of concrete impact problems. A notable feature of many EPIC reports is that a number of the computational options available in the code are exercized in a given problem. Since calculations are typically available showing Lagrange calculations alone, Lagrange with SPH coupling, SPH alone, etc, it becomes much easier to separate the effects of computational devices from constitutive behavior. The results obtained depend very much on how well the concrete model has been calibrated. *This holds true of all code calculations*. Recall that none of these codes can be used at present to predict *a priori* the behavior of concrete under short-term loading. recall further that for most tests, comparisons are made for peak deflections, degree of damage, hole peenetration depth and crater diameter. These are global parameters obtained by multiple integration of the model equations. Integration is a smoothing process. Hence, such comparisons are poor discriminators of constitutive equation behavior. they are necessary, but not sufficient, conditions to determine code efficiency.

The Use of DYNA for Simulations of Concrete Behavior

DYNA has become the Lagrangian code of choice for short-duration loading simulations, not only for metals but other material as well. Its popularity for concrete simulations is attested to by the large number of papers appearing in the list above. Part of the reason for DYNA's popularity lies in the large choice of material models available in the code. Another is a built-in library of constants for the various constitutive models. Thus, concrete impact calculatiions can be done by novices by simply selecting the crushable material model and specifying a few additional parameters such as yield strength and tensile cutoff. The results will be as good as the constants in the material model. Another factor will be the impact speed. At low impacts, sufficient time exists for microcracks to form and coalesce. As velocity increases, pressure increases and the target's response time decreases. Inertia dominates the problem. For high enough velocities, a simple integration of the momentum equation coupled with a rudimentary material description can produce results every bit as good as running a wave propagation code. In this loading regime, both approaches are the same but the code does more work. In good hands, DYNA produces results comparable with other computational techniques and agreement with experiment for global parameters.

Versions of DYNA are maintained both at Livermore Software Inc. (John Hallquist) and at Lawrence Livermore National Laboratory.

The Weidlinger Model

The model developed and used by Weidlinger Associates is a CAP model (see Appendix A for a generic CAP model descriptions). The workings of this viscoplastic, three-invariant isotropic softening model and its application to concrete are explained in Tennant et al (1997). The model is implemented in a proprietary code and has been applied to the study of complex structures undergoing complex loading (aircraft impact, blast and impact loading of structures, etc). As Weidlinger scientists are usually

meticulous in determination of input parameters and show a keen understanding of the underlying physics, code calculation results usually echo experimental results quite well. Working with a code that was developed in-house is another advantage. The users are well aware of its characteristics and eccentricities.

Other Related Efforts

Some very valuable experiments measuring radial stress during the penetration of concrete by 50 mm diameter steel projectiles have been performed by Gran and Frew (1997). The test data provide peak and residual stresses as functions of radial and axial position. This is extremely valuable data that can be used to evaluate not only wave propagation codes but other prediction methods as well.

Clegg et al (1997) have used the SPH (Smoothed Particle Hydrodynamics) technique in AUTODYN-2D to look at penetration into multi-layered soil and concrete targets. The SPH calculations produced results similar to those obtained by pure Lagrange or pure Euler methods.

Hiermaier and Thoma (1997) compared and SPH code with the results of AUTODYN and a local code using a finite element description of the projectile coupled with a force law used to calculate the projectile trajectory. A mixed bag of equations of state, including Mie-Gruneisen and porous, as well as the Johnson-Holmquist strength model, were used. This mixture of constitutive models almost results in making apple to oranges comparisons. Nevertheless, the ersatz code and AUTODYN come within 15% of the experimentally determined penetration depth while the SPH code is off by 30%.

Heiser and Murek (1997) used AUTODYN and DYNA to study perforation and damage in plain concrete. They compared results obtained with Mohr-Coulomb and Holmquist-Johnson-Cook constitutive models. A porous material equation of state was used. Striking velocities were below 1100 m/s so that the projectile was not deformed. At the lowest impact velocities (300 - 400 m/s), all models performed poorly, some overestimating residual velocities by as much as 100%. As velocity increases and inertia dominates, results improve dramatically. Depending on the model, residual velocity predictions deviate from experiments by 0 - 20%. Deviation at low impact velocities ranges from 0 - 32% while at higher (650 m/s) velocities the range is 0 - 35%.

Summary

The general feeling in the community, as stated in numerous publications, is that structural dynamics codes and wave propagation codes have matured considerably, but cannot as yet be used to predict a priori the behavior of concrete and concrete structures to short-duration loading. This is due to the deficiencies of the constitutive models available at this time as well as a paucity of high rate data for a wide enough sampling of concrete.

The two most commonly used models are the visco-elastic-plastic models, sometimes referred to as CAP models, as well as crushable material models based on Herrmann's 1969 p-alpha model. These models carry several underlying assumptions, to wit, that the material is

homogeneous
isotropic
ductile

Static data would indicate that concrete is none of these, so why even consider such models? One answer is that they exist and can be adapted to mimic the behavior of concrete under certain circumstances. Inertia exists among humans as well as inanimate objects and adaptation is a lower energy mode than development. Hence the propensity to adapt models which have worked well for other applications to concrete. Another is that, after suitable curve fitting with the relatively few parameters in these models, results are obtained which resemble experimental observations. Individual cracks and their propagation cannot be modeled with any degree of accuracy but the overall damage zone, the projectile residual velocity, the hole size or penetration depth can be determined at least to within 30% of experiments, sometimes closer. After looking at close agreement between calculations and experiments it is not difficult to convince oneself that at high strain rates concrete does indeed exhibit plastic behavior. However, the high rate data and definitive instrumented experiments do not exist to allow a definitive answer to the question of concrete plasticity at this time.

In a way, this question is analogous to the situation which existed for metals some 30 - 40 years ago. Lacking a definitive data base and corroborating experiments, code users tweaked most available free parameters in the strength, failure and pressure models of the day. Only gross deformation data (displacements and velocities) were available then. When, after some tweaking, code results finally coincided with experiments, a shout of Eureka! went up, together with a not too suble claim that the computer model must resemble reality. Another twenty years went by before the accumulation of dynamic data and improved constitutive models of popular structural materials such as steel, aluminum and later high density materials showed that the initial Eureka! was a bit premature.

Some high rate data exists, but not nearly enough. What does exist is scattered in journal articles, company or government reports and conference proceedings. Hugoniot data - the response of concrete to shock loading - is virtually nonexistent. Very little is available on the behavior of yield and failure with increasing pressure. Thus, the situation is this: no definitive data - no definitive computational models. At this time, the running of a large wave propagation code to determine short-duration response is an exercize in curve-fitting. It may add some details not available from experiment, but extrapolation to other cases is both impossible and dangerous.

For some appplications, codes may not be needed. Smith (1997), in discussing terrorist bombings and preventive measures states:

In the overall scene of devastation it is the writer's view that it is hardly justifiable to attempt to predict by dynamic analysis the complete response of all the building elements to a terrorist bombbecause

1. The bomb size and position are not known in advance. It is in fact also difficult to ascertain bomb size after the event, when the physical effects are observable!

2. The detailed response of most individual building components is not important. A general awareness of the overall expectation of damage level is sufficient for a building owner or occupier to decide whether to make extra provisions.

3. It is the writer's view that in the scenarios where the structural survival of the frame is not in question, the prime objective should be to keep all, or most of the blast pulse out of the building, thus preserving life and injuries, and allowing business hardware to remain functional or at least to be retrievable. If the integrity of the facade and glazing is preserved, the internal environment remains clean and safe. It is therefore in the perfon-nance of glazing and facades that research can most profitably be done.

4. Lastly, the amount of detail and effort needed to accurately model all the building components to reflect the true dynamic response behaviour is extremely expensive, labour and time consuming. The pictorial and descriptive records of previous incidents in fact already do the job. Programmes based on iso-damage curves are in effect also simply referring to previous observations, and are as reliable as the data they are based on.

In short, in terms of applications to concrete, structural dynamics and wave propagation codes are research tools which hold considerable promise for the future but currently cannot serve as predictive tools. This is not due to the numericas of the codes but to deficiencies in curent constitutive models for the short-duration loading and response of concrete and concrete structures.

Recommendations

Long-Term Efforts

(a) The development of constitutive models for concrete will advance only insofar as material characterization at high strain rates and under high pressures advances. Two things are required:

1. the generation of a data base of strength and failure data for plain and reinforced concrete at strain rates of 0.01/s through 1000/s.

2. the determination of shock Hugoniots for concrete, together with determination of the variation of strength with increasing pressure

At first, this data would most likely be in tabular form. Once a sufficiently large data base is developed, models should be developed to put the data in functional form for use in analytical and numerical models.

The information on high rate behavior of concrete should be gathered under one roof - a handbook, a database - and kept current by regular updates, say every 2 - 3 years, depending on activity in the field.

(b) Once sufficient data is available, efforts should be made to develop constitute models for concrete that are physics-based. Once incorporated in analytical models or computer codes, the predictions for standardized problems should be tested against instrumented experiments which are sufficiently detailed to allow evaluation of both local and global variables. To this end, test methods using time-resolved surface measurements, in situ measurements and any other techniques which may become available should be developed.

It is strongly recommended that, in order to accomplish these goals, funding for such projects be regularized. Far better results will be obtained if such efforts are funded regularly at a level of, say, $100,000 per annum for, say, 10 years than if the projects are allowed to languish for 9 years and then funded for $1,000,000 for one year. Small but consistent levels of funding are the best way to advance the state of the art.

Short Term Projects

To properly evaluate the dynamic behavior of concrete, a combination of empirical data, simple analytical models and occasional code calculations are needed. To this end, an array of simple models, such as the rigid body-spring models or some combination of finite differences or finite elements with analytical models or loading functions should be developed. These run in minutes on modern desktop computers and, together with experimental results and emprirical data, allow rapid parametric studies to scope out a problem and get a feeling for the magnitudes of key variables involved. These can then be checked with code calculations or more advanced models. If needed, material characterization or additional modeling with a refined model can be undertaken, and this procedure iterated until a satisfactory design is obtained.

An engineer with only raw data and a computer code at his disposal is lost, given today's state-of-the-art.

The conference proceedings mentioned at the beginning of the report should be obtained. A point should be made to send at least one or two engineers to these conferences as they occur. This is an excellent way to keep up with the state of the art and meet the principal players. Adavantage should also be taken of short courses that are

available in this area. The one on ***Modern Protective Structures***, mentioned previously, is especially valuable for developing an appreciation of the subject and getting hands-on practise with empirical models.

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APPENDIX A

Numerical Modeling of High Velocity Impact in Non-Metallic Materials

Y. D. S. Rajapakse and J. R. Vinson (eds.)
HIGH STRAIN RATE EFFECTS ON POLYMER, METAL AND CERAMIC MATRIX COMPOSITES AND OTHER ADVANCED MATERIALS
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NUMERICAL MODELING OF HIGH VELOCITY IMPACT IN NON-METALLIC MATERIALS

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ABSTRACT

Materials and structures subjected to fast, transient loading (e.g. vehicular impacts, blast and impact loading of structures, the response of containment structures to internal and external stimuli, among others) share several common features which makes them tractable by a set of numerical tools frequently referred to as wave propagation codes or hydrocodes. The successful simulation of such phenomena requires adequate spatial resolution, a constitutive model which incorporates the features observed experimentally and data for the constitutive model appropriate to the range of strain rates encountered in reality. This paper focuses on the available constitutive models for material deformation under high rate loading as well as sources of high rate data. It is found that for metallic materials, adequate models and data exist to model large deformation effects quite accurately for diverse applications. However, failure models range from inadequate to impractical and the available data for existing models is very limited. For non-metallic materials, both definitive computational models and an adequate data base for high rate loading simulations are lacking. Much of the current work in this area relies on ad hoc models, some of which lack a firm physical foundation.

BACKGROUND

Explosions result from the sudden release of energy. The energy may come from an explosive, wheat flour dust in a grain elevator, pressurized steam in a boiler or an uncontrolled nuclear transformation. The accumulated energy is dissipated in various ways such as in blast waves, the propulsion of missiles or by thermal or ionizing radiation (Kinney and Graham ,1985). As the blast wave travels away from its source, the overpressure (pressure in excess of atmospheric pressure) at the front steadily decreases and the pressure behind the front falls off in a regular manner. After a short time, the pressure behind the front drops below atmospheric so that an underpressure rather than overpressure exists. During this negative (rarefaction or suction) phase, a partial vacuum is produced. Air is sucked in instead of being pushed away. At the end of the negative phase, which is somewhat longer in duration than the positive phase, the pressure returns to ambient. Underpressures usually have a magnitude of less than 28 kPa whereas overpressure can exceed 2 MPa.

Structures suffer damage from air blast when overpressures exceed 3.5 kPa. The distance to which the overpressure level extends depends on the energy yield of the explosion. The blast loading is seen as a lateral dynamic pressure, applied rapidly, lasting for a second or more and continuously decreasing in strength. The factors which most strongly affect the structural response are the inertia of the structure, as measured by the mass and strength of the structure, the overall structural design and the ductility of the materials and members comprising the structure. The strength of the structure is not only a material characteristic but includes the massiveness of the

construction and redundancy of supports. If the strength is not isotropic, then the orientation of the structure with respect to the burst becomes important (Kinney and Graham ,1985; Glasstone and Dolan ,1987; Bailey and Murray ,1989).

Impact, by contrast, is a localized phenomenon. Upon collision with a target, the process of transforming some or all of the kinetic energy of the striker into deformation and failure of the colliding materials is governed by wave propagation. On contact, a compression wave propagates into both striker and target with an initial intensity of $\rho c v$, where c represents sound speed, a material characteristic, and v is the particle velocity. Because of geometric effects, this initial intensity rapidly decays to a value of $v^2/2$. The presence of free surfaces and material interfaces causes multiple reflections of the propagating waves causing the occurrence of sharp compressive and tensile stress gradients. Depending on the duration of the tensile pulses and their intensity, material failure in the form of physical separation of material may occur by a number of mechanism (see Curran, Seaman and Shockey ,1977; Backman and Goldsmith ,1978; Johnson ,1972; Zukas et al ,1982; Zukas ,1987; 1990; Bushman et al ,1993; Blazynski ,1987 and Jones ,1989 for comprehensive discussions on impact phenomena).

The stress waves in the colliding solid propagate at a characteristic velocity that is a property of the material. The particle velocity (the speed at which material particles are actually displaced in the solid), however, governs the ensuing deformation. Because of this, coupled with the presence of multiple wave reverberations due to geometric effects, the most intense deformations occur within 3-6 characteristic striker dimensions (e.g., 3-6 diameters if the striker has regular geometry such as a cylinder, sphere or cone). Within this region, strain rates in excess of 10^5 are not uncommon, as are plastic strains exceeding 60% and pressures well in excess of the material strength. Loading and response times are in the submillisecond regime. Unlike structural response problems, the high frequency components of the structure are excited and lead to material failure by a wide variety of mechanisms.

Because of the localized nature of the impact processes (Wright and Frank ,1988), the geometries that need to be considered can be somewhat simplified. Unlike the structural dynamics problem, catastrophic failure by penetration or perforation can occur long before the distant boundaries have any effect. Thus, boundary conditions tend to be very simple, allowing either for perfect transmission or reflection of the incoming signal. Initial conditions most often specify the velocity of the striker, although these can be augmented by surface pressures or shears depending on the problem.

When it becomes necessary to consider the lethal effects of debris caused by blast loading, the blast and impact problems share several common features. Wave propagation effects must be accounted for explicitly since the interaction of stress waves with free surfaces, material interfaces and geometric discontinuities lead to material failure such as spallation or fragmentation. Inertia effects must be accounted for in considering both types of problems. This implies the use of explicit time integration techniques in numerical analyses as well as contact-impact (sliding interface) logic to properly account for momentum distribution between flying debris and surrounding structures. Both types of problems need constitutive models for deformation and failure at high strain rates. The "material data" or constants which are embedded in these models must also be determined from wave propagation experiments, i.e., at strain rates appropriate to the problems being analyzed. Hence, the early stages of a problem in blast loading and most high velocity impact problems can be successfully analyzed by wave propagation codes, often referred to as hydrocodes.

COMPUTATIONS INVOLVING METALLIC MATERIALS

Modern hydrocodes decouple the material response for metallic materials into a volumetric and a deviatoric part

The volumetric or hydrodynamic response is obtained through an equation of state. By far the most popular equation of state (EOS) in existing codes is the Mie-Grueisen equation of state, which makes use of Hugoniot data obtained from plate impact experiments and includes an energy term to reach off-Hugoniot states. The Mie-Grueisen EOS assumes that the material will respond as a solid throughout its loading regime. For transitions from solid to vapor or liquid states, another equation must be used. The Tillotson EOS has been very popular for many years for problems in hypervelocity impact where such transitions occur with high frequency. Examples of equations of state in modern hydrocodes are shown in Table 1. Extensive compilations of equation of state data exist, due to Van Thiel (1977), Marsh (1980), Dobratz (1981),

Table 1: Common Equation of State Formulation in Hydrocodes

EQUATIONS OF STATE

MIE - GRÜNEISEN:

$$P = P_H\left(1-\frac{\Gamma\mu}{2}\right)+\Gamma\rho\,(I-I_0)$$
$$\mu = \rho/\rho_0 - 1$$

TILLOTSON:

FOR $\rho > \rho_0$ AND $0 \le I \le I_s$

$$P = P_\pi + A\mu + B\mu^2$$
$$P_\pi = I\rho\left[a+\frac{b}{I/(I_0\eta^2)+1}\right]$$

FOR $\rho < \rho_0$ WITH $I > I_s$

$$P = aI\rho\left[\frac{bI\rho}{(1+I/(I_0\eta^2))}+A\mu e^{-\beta(1/\eta-1)}\right]e^{-\alpha(1/\eta-1)^2}$$
$$\eta = \rho/\rho_0$$

LOS ALAMOS:

$$P = \begin{cases}\left[A\mu+\rho_0 I(B+\rho_0 IC)\right]/(\rho_0 I+\phi_0) & \text{IF } \mu \ge 0\\ \left[\mu A_1+\rho_0 I(B_0+\mu B_1+\rho_0 IC)\right]/(\rho_0 I+\phi_0) & \text{IF } \mu \le 0\end{cases}$$

WILKINS:

$$P = a\eta^\alpha + b\left(1-\frac{\omega}{R}\eta\right)e^{-R/\eta}+\omega\eta I$$

GAMMA LAW:

$$P = (\gamma-1)(1+\mu)I$$

JONES, WILKINS, LEE (JWL):

$$P = A\left(1-\frac{\omega\eta}{R_1}\right)e^{-R_1/\eta}+B\left(1-\frac{\omega\eta}{R_2}\right)e^{-R_2/\eta}+\omega\eta I$$

Kohn (1969), Vigil (1989) and others. For situations where EOS data does not exist, there are at least a dozen methods by which it can be obtained. Thus, years of experimentation and systematic collection of shock wave data have made computation of the hydrodynamic response of materials almost a routine matter. Such is not the case when considering material strength, or deviatoric, effects.

An incremental elastic-plastic formulation is used to describe the shear response of metals in present finite-difference and finite-element codes. The plasticity descriptions are based on the assumed decomposition of the velocity strain tensor, $\dot{\epsilon}$, into elastic and plastic parts

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^p$$

together with incompressibility of the plastic part

$$\dot{\epsilon}^p_{11} + \dot{\epsilon}^p_{22} + \dot{\epsilon}^p_{33} = 0$$

The von Mises yield criterion is typically used to describe the onset of yielding. Provision is made to account for strain hardening, compressibility and thermal effects but more often than not such data is not available for practical calculations. Earlier hydrocodes incorporated the Jaumann stress rate. Because of difficulties with this formulations at strains exceeding 40% (Walters and Zukas ,1989) , this formulation is gradually being replaced with alternatives (Green and Naghdi ,1965); Dienes ,1979). The typical constitutive model for metallic materials is depicted in Table 2.

Dynamic events often involve increases in temperature due to adiabatic heating. To accurately predict the response of a material, the effect of temperature on the flow stress must be included in a constitutive model. An attempt to account for the effects of strain, strain rate and temperature is made in the model proposed by Johnson and Cook (1983; 1985). This is an empirical model involving five constants A, B, C, n and m. The functional form

$$\sigma = [A + B\epsilon^n][1 + C\ln\dot{\epsilon}^*][1 - (T^*)^m]$$

where

$$T_* = \frac{(T - T_{room})}{(T_{melt} - T_{room})}$$

and

$$\dot{\epsilon}^* = \frac{\dot{\epsilon}}{\dot{\epsilon}_0}, \quad \dot{\epsilon}_0 = 1.0/s$$

of the model constitutes the authors' best guess as to how material behaves under high rate loading. It is not based on any theory. The constants must be determined from experiments. Both the experiments and the model decouple the effects of the parameters which enter into it. One result is the implication of the model that strain rate sensitivity is independent of temperature, a feature that is not generally observed for most metals (Nicholas and Rajendran ,1990). In fact, rate sensitivity is found to increase with increasing temperature while flow stress decreases.

Zerilli and Armstrong (1987) used dislocation dynamics concepts which also accounts for strain, strain rate and temperature effects in a coupled manner. For f.c.c. metals their model takes the form

$$\sigma = C_0 + C_2\epsilon^n[e^{(-C_3T + C_4T\ln\dot{\epsilon})}]$$

while for b.c.c metals it is given as

$$\sigma = C_0 + C_1[e^{(-C_3T + C_4T\ln\dot{\epsilon})}] + C_5\epsilon^n$$

where σ represents stress, ϵ and $\dot{\epsilon}$ represent strain and strain rate, respectively, T is the temperature and the C's are material constants. The rationale for the difference in the two forms is in the stronger dependence of yield stress on temperature and strain rate is known to result for b.c.c. metals than f.c.c. metals. The constants for the model have been determined for only two materials. A comparison was made between the Johnson-Cook and Zerilli-Armstrong models for Taylor cylinder impact experiments (Johnson and Holmquist ,1988) . The Zerilli-Armstrong model gave slightly better correlation with experimental results although, being small strain models, neither was very accurate in describing large strain behavior

Table 2: Representative Constitutive Model for Metals

CONSTITUTIVE EQUATIONS

ELASTIC-PLASTIC MATERIAL:

(a) STRESS DEVIATORS

$$\dot{s}_{ij} = 2G\left(\dot{\epsilon}_{ij} - \frac{1}{3}\delta_{ij}\,\dot{u}_{k,k}\right)$$

(b) VELOCITY STRAINS

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\dot{u}_{i,j} + \dot{u}_{j,i}\right)$$

$$\dot{\omega}_{ij} = \frac{1}{2}\left(\dot{u}_{i,j} - \dot{u}_{j,i}\right)$$

(c) JAUMANN STRESS RATE

$$\sigma_{ij} = \sigma_{ij} + \sigma_{im}\,\omega_{mj} - \omega_{im}\,\sigma_{mj}$$

(d) TOTAL STRESS

$$\sigma_{ij} = s_{ij} - \delta_{ij}\,P$$

$$s_{ii} = 0 \qquad \sigma_{ii} = -3P\,(\rho, 1)$$

(e) VON MISES YIELD CRITERION

$$s_{ij}\,s_{ij} \le 2/3\,Y^2$$

IF YIELD STRESS EXCEEDED:

$$s_{ij} = s_{ij}\left[\frac{2Y^2}{3s_{ij}\,s_{ij}}\right]^{1/2}$$

(Nicholas and Rajendran ,1990).

The Bodner-Partom model (1975) is based on dislocation dynamics concepts and treats strain rate and temperature effects in a coupled manner to more realistically account for observed behavior. It has been used in analysis of rate history effects (Bodner and Merzer ,1978), propagation of plastic waves in long rods (Bodner and Aboudi ,1983) and precursor decay analysis (Nicholas, Rajendran and Grove ,1987a;b). The Bodner-Partom Model is shown in Tables 3 and 4. Procedures for determining the constants are given by Rajendran (1994).

Models developed by Steinberg, Cochran and Guinan (1987) and Steinberg (1987; 1991), assume strain rate saturation and allow variation of shear modulus and yield strength with pressure and temperature:

$$G = G_0[1 + [\frac{G_P'}{G_0}]\frac{P}{\eta^{1/3}} + [\frac{G_T'}{G_0}](T - 300)]$$

$$Y = Y_0(1 + \beta\epsilon)^n[1 + [\frac{Y_P'}{Y_0}]\frac{P}{\eta^{1/3}} + [\frac{Y_T'}{Y_0}](T - 300)]$$

Steinberg (1987) catalogs the available parameters for the model.

Other models which account for the variation of strength with strain, strain rate, temperature, pressure or other parameters are discussed in the literature (Nicholas and Rajendran ,1990; Rajendran ,1994; Follansbee and Kocks ,1988; Meyer ,1992). These are generally one-dimensional models and have been used to interpret various experiments involving high rate behavior. In principle, one-dimensional models can be generalized to three dimensions through the use of effective stress and strain formulations. Except for isolated incidences to support specific laboratory experiments, these have not found their way into production computer codes and will not be mentioned further.

Material failure is treated as an instantaneous process, based on maxima or minima of field variables (see Zukas ,1987 for detailed descriptions of hydrocode failure criteria and post-failure models). It is well known at this time that material failure is a time-dependent phenomenon. However, the degree of material characterization required for advanced failure models taking into account nucleation, growth and coalescence of microvoids and cracks to form free-flying fragments is so expensive that it is not used in practice. Simple, one-parameter failure criteria are known to be inadequate descriptors of material failure at high strain rates. However, they frequently provide answers which are close enough for all practical purposes and the parameters for such models can frequently be obtained from existing high-rate data.

The above formulations are virtually common to all wave propagation codes. In addition, some codes also allow for a number of alternative material descriptions: elastic (isotropic or orthotropic); piecewise linear elastic-plastic models with strain hardening effects; linear viscoelastic; thermo-elastic; thermo-plastic with material properties temperature-dependent. In addition to the von Mises flow rule, general isotropic, kinematic and combined strain hardening rules allow yield surface translation as well as expansion or contraction.

It is generally agreed (Immele, 1989) that hydrocodes are a vital supplement to impact experiments. Computations generally return more insight and data than tests; allow generalizing the test data to assess the effects of variables [e.g., strength, detonation velocity], that had not been tested and permit parameter variations and tests of sensitivity. Organizations making extensive use of numerical simulations find that the number of tests can often be reduced and that fewer surprises are encountered during testing, all resulting in reduced costs and manpower demands (Herrmann ,1977, Immele ,1989). The single biggest drawback to the accuracy of code predictions is the primitive status of the failure models (NMAB ,1980; Zukas ,1987). If inertia effects dominate material or structural response, code predictions can be quite accurate - with 3 - 5% of experimentally observed deformations, penetration depths and residual velocities. When brittle or ductile failure modes dominate material response, code results are very likely to be wrong in both qualitative and quantitative predictions of impact events.

COMPUTATIONS INVOLVING NON-METALLIC MATERIALS

A large number of practical problems involve impact or impulsive loading of structures consisting primarily of non-metallic materials. Reactor containment structures must withstand external impacts of commercial aircraft as well as tornado-borne debris such as automobiles, telephone poles, rocks and other

Table 3: Bodner - Partom Model

$$\dot{\epsilon}_{ij}^{p} = D_0 \, e^{\left(-\frac{(n+1)}{n}\left(\frac{Z^2}{3J_2}\right)\right)^n} \frac{S_{ij}}{\sqrt{J_2}}$$

$$\dot{Z} = m\,(Z_1 - Z)\,\dot{W}_p$$

D_0 is the limiting strain rate, n is strain rate sensitivity parameter
m is strain hardening parameter
Z_0 and Z_1 are initial and saturated values of the internal state variables Z
Z describes resistance to plastic flow, and loading history dependency.
n is a temperature dependent constant. W_p is plastic work.

Table 4: Bodner - Partom Model Constants

Material	Z_0 (GPa)	Z_1 (GPa)	n	m_0 GPa^{-1}	m_1 GPa^{-1}	α GPa^{-1}	A	B
C1006 Steel	5.5	7.0	0.4	15	0	0	0.245	45
HY100 Steel	2.4	3.6	1.2	10	0	0	NA	NA
1020 Steel	0.64	0.93	4.0	30	0	0	NA	NA
MAR-300 Steel	2.2	2.4	4.0	5	0	0	NA	NA
Armco Iron	2.65	4.2	0.58	56	0	0	NA	NA
OFHC Copper	0.8	6.6	0.4	11	150	1500	NA	NA
6061-T6 Aluminum	0.45	0.55	4.0	120	0	0	-2.86	2343
7039-T64 Aluminum	0.56	0.76	4.0	28	0	0	NA	NA
Pure Tantalum	1.3	3.1	0.74	20	0	0	NA	NA
W-2 Tungsten	8.75	10.0	0.58	150	0	0	0.186	134
Nickel 200	0.22	0.82	4.0	40	0	0	NA	NA
MAR-250 Steel	2.5	2.7	5.0	20	0	0	NA	NA
AF1410 Steel	2.4	2.75	5.0	15	0	0	NA	NA

NA – The high temperature constants are "Not Available"

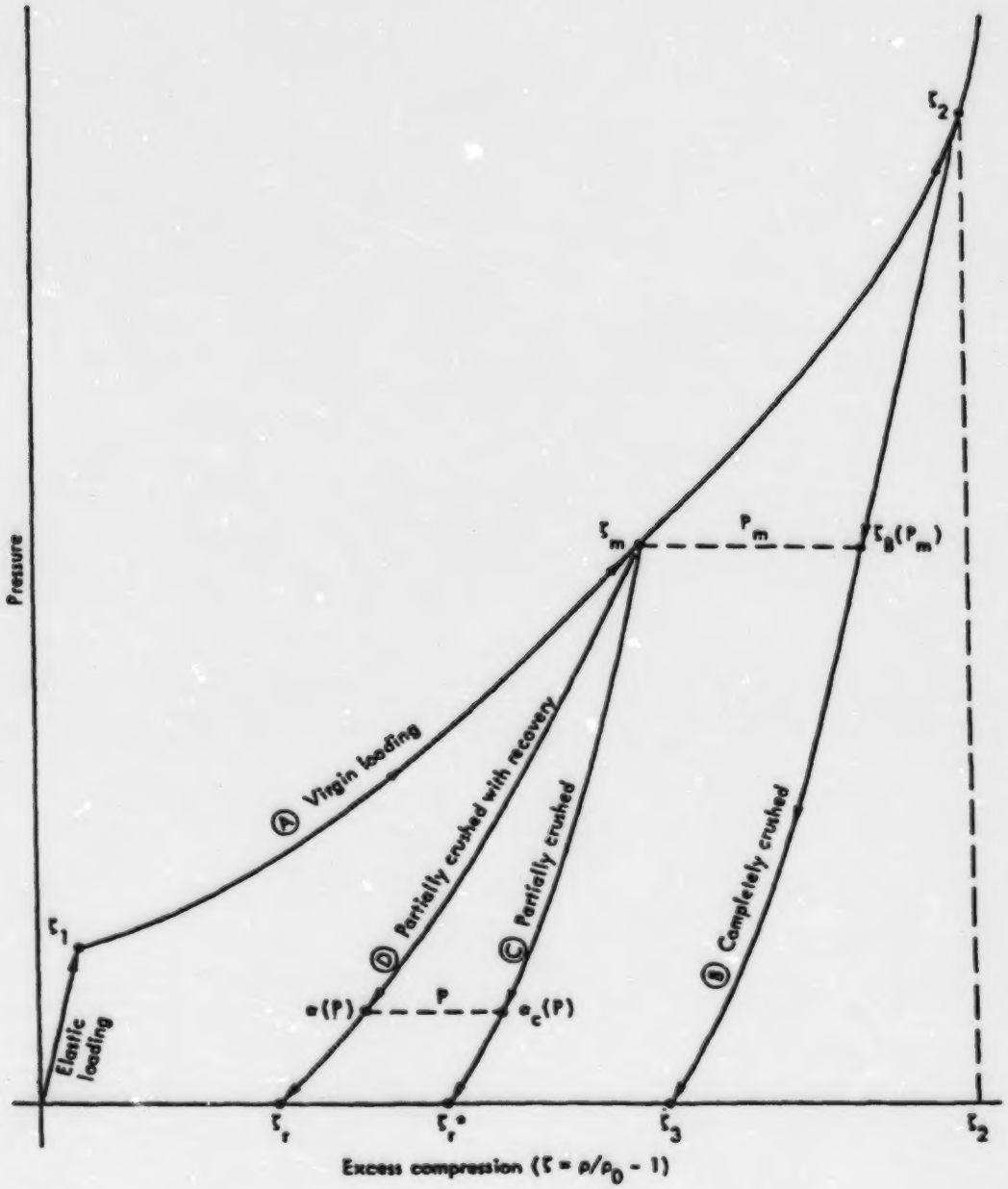

Figure 1: Constitutive Model for Porous Media

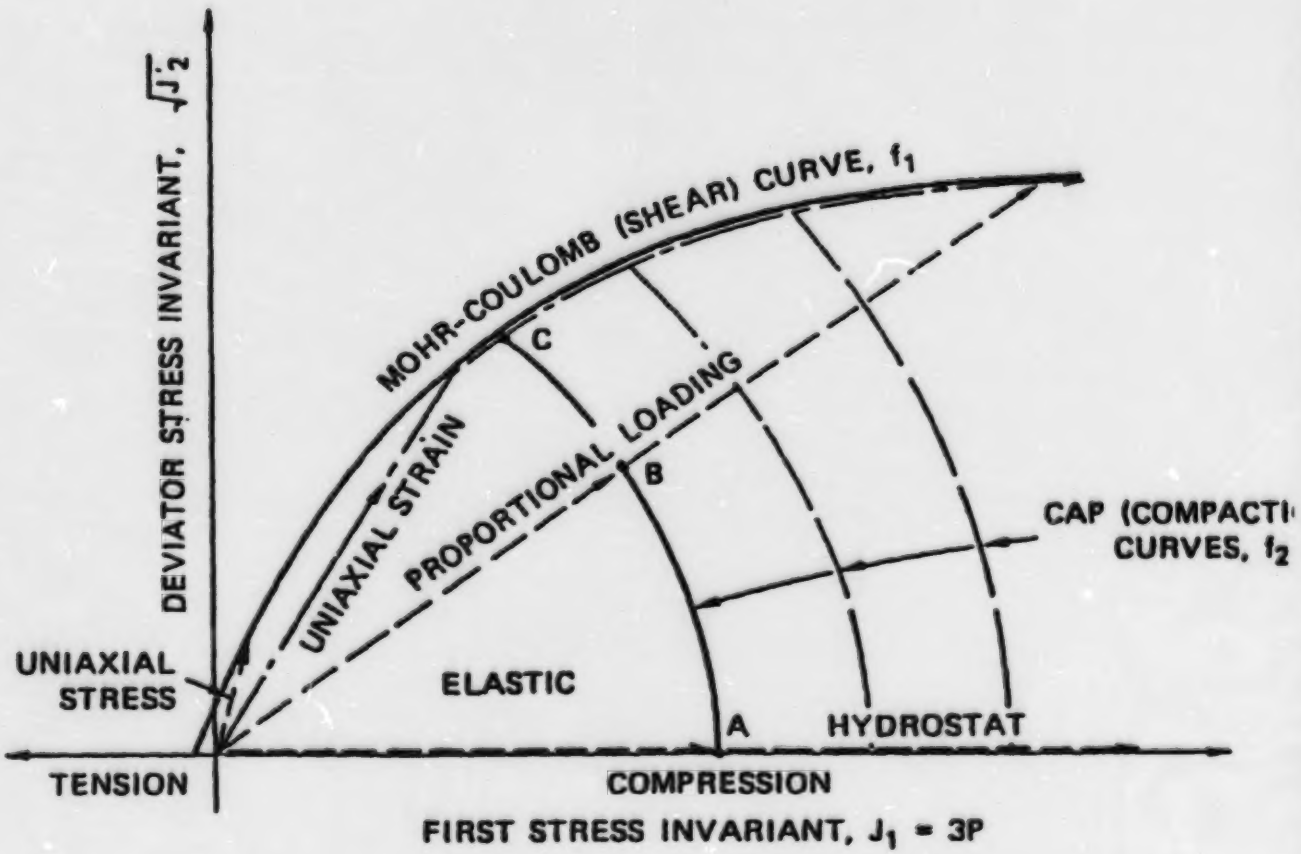

Figure 2: CAP Model

readily available natural and manmade materials. Such structures are typically made of reinforced concrete or layers of concrete and steel. Protective bunkers for personnel or storage facilities for energetic materials are often made of layers of metal, concrete and soil. Body armor consists of multiple layers of Kevlar or nylon. Impacts onto sand, water, ice, snow and various geological materials are not uncommon in everyday applications. Ceramics are being used in many industrial and military applications. Finally, there is a very large class of composite materials that is slowly beginning to take the place of metals, if not for ballistic protection, then for safety considerations. Perforation of a metallic fuel tank on an aircraft may cause failure through a hydrodynamic ram effect. However, if the aircraft survives that, sparks caused by penetration of a metallic tank by a metallic striker can cause explosion of the fuel. Replacement of the fuel tank and other aircraft components by composite materials with strengths similar to that of metals will not improve ballistic performance but will inhibit sparks (Held, 1993).

What, then, is the status of computations with such materials? Thoma and Vinckier (1993) summarize the situation very nicely: "...whereas the calculations of high velocity impact on isotropic materials can be done on a routine basis, the simulation of the impact and penetration process into nonisotropic materials like reinforced concrete or fiber-reinforced materials is still a research task". This state of affairs results from three conditions:

a. lack of definitive computational models

b. lack of high strain rate data for both volumetric and deviatoric response for anisotropic materials

c. lack of an experimental data base resulting from controlled, instrumented impact experiments to serve as a basis for code validation.

The first of these issues is probably the easiest to resolve. A short description of constitutive models for nonmetals is given next. Ceramics are specifically excluded here since they are the topic of another presentation at this symposium.

Soils and crushable (porous) materials are frequently represented by a p-alpha model first developed by Herrmann (1969). A variant of this formulation is incorporated in the NIKE code (see Zukas ,1990 for a review of modern hydrocodes). ADINA includes a concrete model with multi-axial material failure envelopes on tension cracking and compression crushing and strain softening; a Drucker-Prager type model for soils and rocks with tension cutoff and compression cap; and a curve (empirical) description soil and rock model with tension cutoff or cracking. A version for plain concrete is included in the EPIC and HULL codes. Burton and Schatz (1975) used such a model in the TENSOR code to study rock crushing.

Data for a p-alpha type model must be determined from individual materials from a number of independent experiments. The generic form of such a model is depicted in Figure 1. The material originally loads along the virgin path A. If the maximum compression is less than ζ_m, unloading also occurs along path A. A totally crushed material would unload along path B. For partially crushed materials the unloading path would be C unless there is elastic recovery, in which case the unloading path becomes D. In order to create such a pressure - compression map, the points ζ_1, ζ_2, ζ_3 and the residual porosity α must be determined through laboratory tests, which can involve the iterative use of computations with an assumed constitutive model and indirect experiments.

Geological materials are commonly handled through a CAP model (Gupta and Seaman ,1979; Wright and Baron ,1979), shown in Figure 2. This has also been adapted for porous materials, concrete and ceramics by judicious adjustment of the various parameters. A non-associative flow rule is commonly used to describe the shear envelope. The cap moves depending on the degree of volumetric response.

Many porous materials yield under shear loading, compact irreversibly under compressive loading, and separate in tension. In concrete and rocks, the shear strength increases with increasing pressure. Also, increasing shear stress reduces the resistance to compaction. This shear-enhanced compaction is common to porous materials. the stiffness of the material also increases with pressure, hence the elastic moduli are functions of the stress state. Thus, CAP models generally include the following:

a. elastic stress-strain relations, including variation of the moduli from initial loading to consolidation

b. Mohr-Coulomb (or shear) curve, including plastic stress-strain relations on the curve

c. CAP curve, including plastic stress-strain relations on the CAP curve and description of the strain hardening of the CAP curve

d. tensile curve, which is the continuation of the Mohr-Coulomb curve into the tension region, stress-strain relation and stress reduction on the curve, and a fracture and separation process.

A comprehensive description of the derivation of a CAP-type model for reinforced concrete as well as experiments to determine the necessary model parameters are given in the report by Gupta and Seaman (1979).

The most complete model for homogeneous orthotropic materials has been incorporated in EPIC. Elastic behavior is determined from the anisotropic elastic stress-strain relationship

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

where C_{ijkl} is specialized to an orthotropic material. The onset of plasticity is determined by the Hill criterion

$$\sigma_{eff} = [\frac{H_1}{2}(\sigma_x - \sigma_y)^2 + \frac{H_2}{2}(\sigma_z - \sigma_x)^2 + \frac{H_3}{2}(\sigma_y - \sigma_z)^2 + 3F_{44}\tau_{xy}^2 + 3F_{55}\tau_{zx}^2 + 3F_{66}\tau_{yz}^2]^{1/2}$$

which reduces to the von Mises law when anisotropy is vanishingly small. The plasticity model also accounts for anisotropic work-hardening. Pressure is determined from either a polynomial equation of state fit to data for the specific composite or estimated from an isotropic Mie-Gruneisen equation of state. Complete derivation of the equations and the numerical algorithm are described Johnson, Vavrick and Colby (1980).

An alternate approach to determination of the high pressure response using mixture theory has been proposed by Anderson, O'Donoghue and Skerhut (1990). Anisotropic constitutive formulations offering minor corrections to the EPIC approach have been proposed by O'Donoghue et al (1992) and Anderson et al (1994).

The anisotropic models in other hydrocodes tend to model only elastic behavior, typically viewing the composite as homogeneous but transversely isotropic or orthotropic. Isotropic behavior is assumed upon the onset of plasticity. For examples see Hallquist (1986), Swegle & Hicks (1979), Key, Beisinger and Krieg (1978), Sedgwick et al (1976), Hallquist and Whirley (1989) to cite but a few.

The above develop models for composites which retain their full load-carrying capability until failure, By and large, however, there is no agreement as to what constitutes failure nor how the material should be treated after failure has occurred. Keep in mind that the above models all assume homogeneous anisotropy. As much or more work remains to be done here regarding failure of composites under high-rate loading as for metallic materials.

In addition, a large number of ad hoc models exist for solids, liquids and explosives. These usually lack theoretical foundation and are little more than attempts to fit existing and incomplete data bases. As many successful calculations with such models can be found as unsuccessful ones. The current interest is in computational models for ceramics and for reinforced concrete. At present, no definitive computational models have emerged. However, this is an area of active research.

PROBLEM AREAS

Assuming that the above models prove adequate in the short run for non-metallic materials - a very dangerous assumption - what about the remaining problems? Where is the data to drive these models. What experiments are available to validate code predictions?

Most of the data available so far comes from traditional high rate test techniques such as the Split Hopkinson Pressure Bar, the Expanding Ring test and plate impact experiments. See Nicholas and Rajendran (1990) for a description of high strain rate test methodology. All such data, however, assesses uniaxial behavior. Biaxial and triaxial evaluation techniques are still in their infancy, incapable of generating on a production basis the data required for routine calculations. The problem can be seen from the following example.

A manufacturer develops a new fiber. A load-elongation curve is determined along the fiber axis but no transverse measurements are made. The fiber is embedded in an isotropic matrix. By alternating layups, a composite structure of some sort is built. At best, a few static measurements are made on the constituents.

Assuming the resulting structure, be it beam, cylinder,

plate or whatever, is orthotropic, 9 constants are required to characterize its elastic behavior. Another 6 are required for the anisotropic yield criterion. At least one parameter is required to characterize work-hardening. Assuming that a one-term equation of state will suffice, another constant is required here. This totals 17 required material descriptors, all determined at strain rates appropriate to the problem at hand. At best, two are available from static tests. Allowing for Divine Intervention, say all 9 parameters for the elastic behavior are available, but from static tests results. Now only 8 more are required, plus an answer to the question of whether static data for composites is a valid descriptor of high rate behavior (our experience with metals indicates that in general the answer will be no).

In order to complete the calculation, the remaining parameters must be determined in some fashion. To assume that this will be done in a logical and consistent manner, following the scientific method, is to place more faith in computational specialists than has been heretofore been warranted. Thus, when evaluating results, it is wise to keep in mind the following comment as quoted by Glenn and Janach (1977): "...we were recently reminded of a comment attributed to the French mathematician, Cauchy, to the effect that, given four parameters, he could draw a credible version of an elephant and given five, he could make its tail wiggle".

Assuming a credible calculation can be completed, how are the results to be displayed? For isotropic materials the important parameters governing short-term response of materials and structures to intense loading are well known. The picture is not as clear for nonmetallic, anisotropic materials. The codes can and do generate volumes of data, but which of the data is significant for particular applications is a matter of judgement. Finally, how will the validity of the calculation be assessed? Comparison of computed results with global parameters such as gross deformations, energies, momenta and quantities related to these are necessary but not sufficient conditions. For metallic materials they are not adequate discriminants of the validity of a constitutive formulation. For nonmetallic materials the case is likely to be worse. For metallic materials, experimental procedures exist for in-situ measurement of arrival times, pressures, wave profiles, strains and other local quantities. For most nonmetallic materials, such experiments remain to be defined.

SUMMARY

Hydrocodes, or wave propagation codes, are a valuable adjunct to the study of the behavior of metals subjected to high velocity impact or intense impulsive loading. The combined use of computations, experiments and high strain rate material characterization has, in many cases, supplemented the data achievable by experiments alone at considerable savings in both cost and engineering man-hours. A large data base exits of high pressure equation of state data. Considerable data on high rate deviatoric behavior exists as well although, unlike EOS data, it is not collected in a few compilations but scattered throughout a diverse literature. Experimental techniques exist for determining either EOS or strength data for materials not yet characterized under high rate loading conditions.

By contrast, computations with non-metallic materials such as composites, concrete, rock, soil and a variety of geological materials are, in effect, research tasks. This is due to several reasons: lack of definitive computational models for high strain rate-temperature-pressure response; lack of a data base for EOS and high rate strength data for such materials; lack of test methodologies for anisotropic materials subjected to high rate loading.

A large number of *ad hoc* models exist for explosives, geological materials, concrete and other non-metallics. Many of these lack a firm theoretical foundation. This is an area where considerable research is required, both to devise appropriate test techniques to measure material response under high strain rates, evelated temperatures and high pressures as well as to develop appropriate constitutive models.

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APPENDIX B

The Concrete Equation of State in HULL

EOS NUMBER 4

THE CONCRETE EQUATION OF STATE

This routine may be called in a purely Eulerian, purely lagrangian or linked run.

The plain concrete equation of state model was developed to provide reasonable response in the 1 to 300 kilobar range. It is based on a very small amount of data, since most available data is at stress levels well below 1 kilobar. However, this routine has been used successfully in scores of calculations of penetrations into concrete at velocities from a few hundred to over 1000 feet/second.

The model was constructed primarily from Hugoniot data on one specific concrete and static yield strength data on several concretes. Assumptions were made for a value for Poisson's ratio, a shear modulus formula, a plastic flow rule and other parameters.

Gregson (REF 10) generated Hugoniot data to stress levels over 500 kilobars for a granite aggregate concrete. The aggregate had an average diameter of 1/8 inch. The initial density varied from mix to mix but averaged 2.185 gm/cc. The solid density had an average of 2.673 gm/cc. The concrete consisted of 18 percent voids, 25 percent granite aggregate, 20 to 25 percent quartz grains and 25 to 30 percent cement paste by volume. The small aggregate size was dictated by his shock-generating method, which consisted of gas-gun launched flyer plates. He employed several measurement techniques to establish stress and velocity. The final data converted to stress vs. excess compression, μ, appears in Figures I-V-2 and I-V-3.

The lowest point in Gregson's data is at 0.75 kilobars. This is a fairly consistent precursor value seen in his stress vs. time data. HULL uses that point to define the density ratio at which significant cracking and void filling occurs.

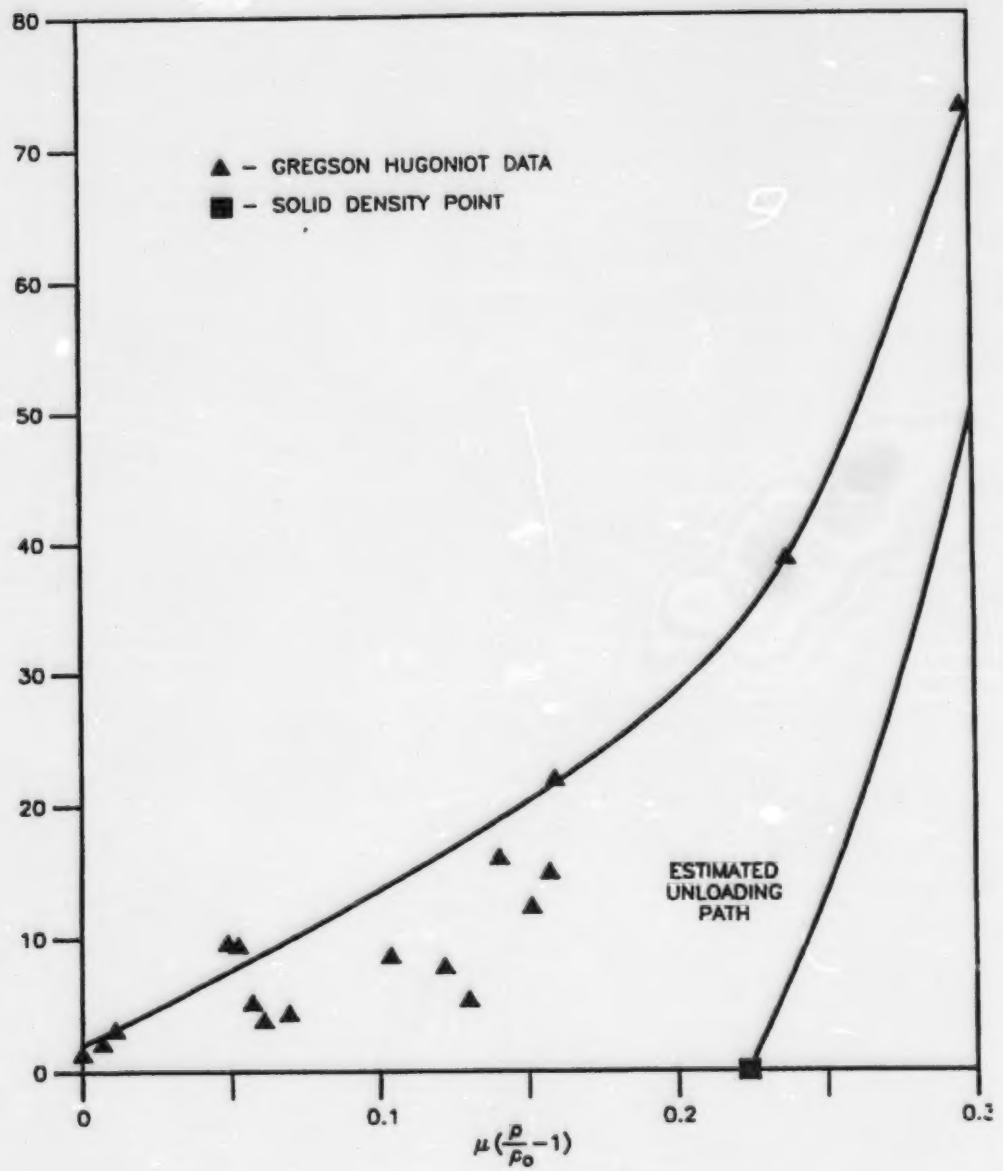

Figure I-V-2. Low Level Hugoniot Stress Data

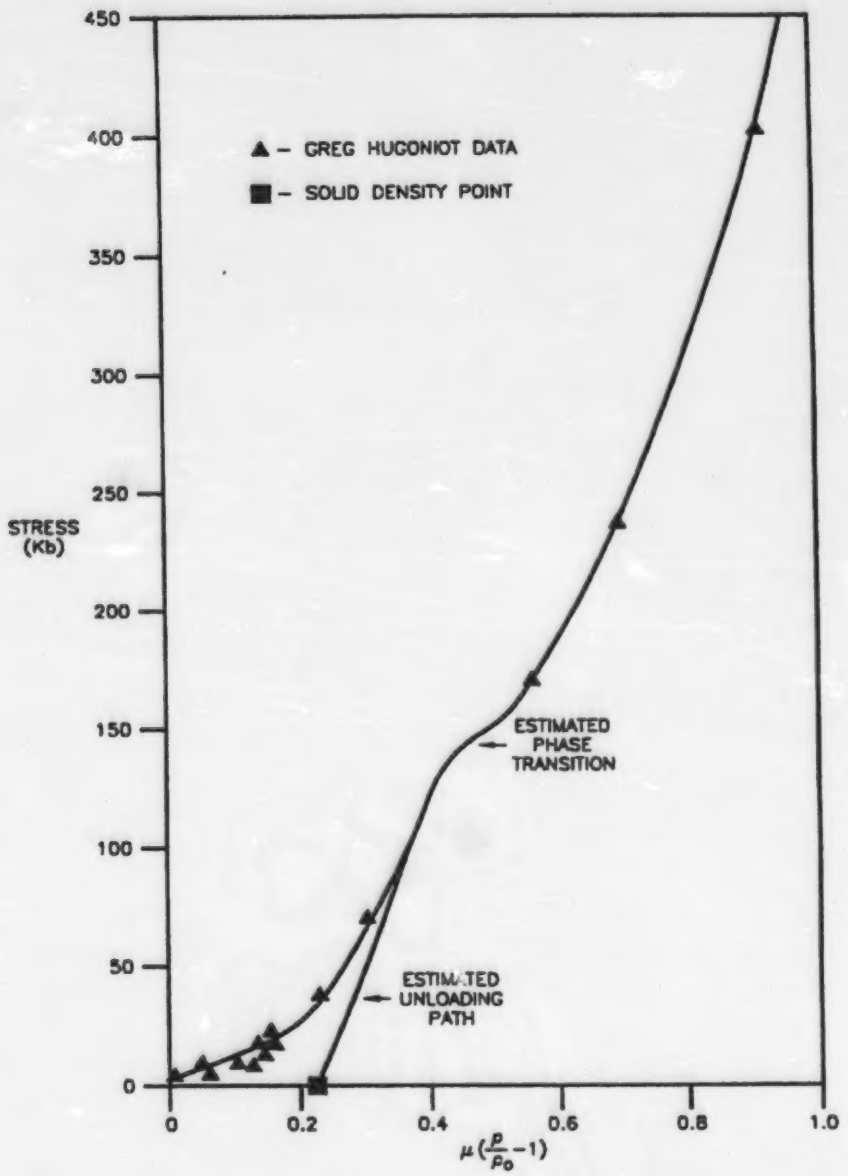

Figure I-V-3. High Level Hugoniot Stress Data

The estimated unloading path on the figures is simply one which allows the concrete to return to its solid density when loaded past the point where all voids are filled. It is a straight line which connects with the loading curve at the total void filled

point with a slope equal to the loading curve at that point. There is no data for this line beyond the solid density point at zero stress.

The yield strength of concrete increases as confining pressure is increased. The only data available at high confining pressures is static data generated by Chinn (REF 11). Chinn's results are shown in Figures I-V-4 and I-V-5 along with some typical low stress level results from McHenry (REF 12). The yield strength and pressure are normalized by f_c', the concrete's unconfined compressive strength. Two theoretical points are shown on Figure I-V-4. One defines failure (flow) at the unconfined compressive strength point, i.e., $Y = f_c'$ at $P = 1/3\ f_c'$. The other is included because a minimum pressure criterion is used to define failure. This minimum pressure was chosen as that occurring in an unconfined tensile test assuming failure when stress is $0.1\ f_c'$ (REF 13). It is numerically pleasing to set $Y = 0$ at this failure point.

The yield strength curves are approximated in HULL with three straight line segments:

$$Y = 0 \qquad \text{if } P \le -0.1f_c'$$

$$Y = 3((P + 0.1f_c')/3)/1.1 \qquad \text{if } (((-0.1f_c')/3) \le P \le f_c'/3)$$

$$Y = P = \frac{2}{3}f_c' \qquad \text{if } ((f_c'/3) < P \le 30f_c')$$

$$Y = 30.67f_c' \qquad \text{if } P > 30f_c'$$

the limit at $P = 30\ f_c'$ is not indicated in Chinn's data. It is included because it is felt there should be a limited value. It is included at the level shown simply because it is not known to exist at a lower level.

It should be emphasized that the yield strength data was developed in static tests. Higher yield strengths may exist at strain rates of interest in conventional munitions work, but data simply does not exist.

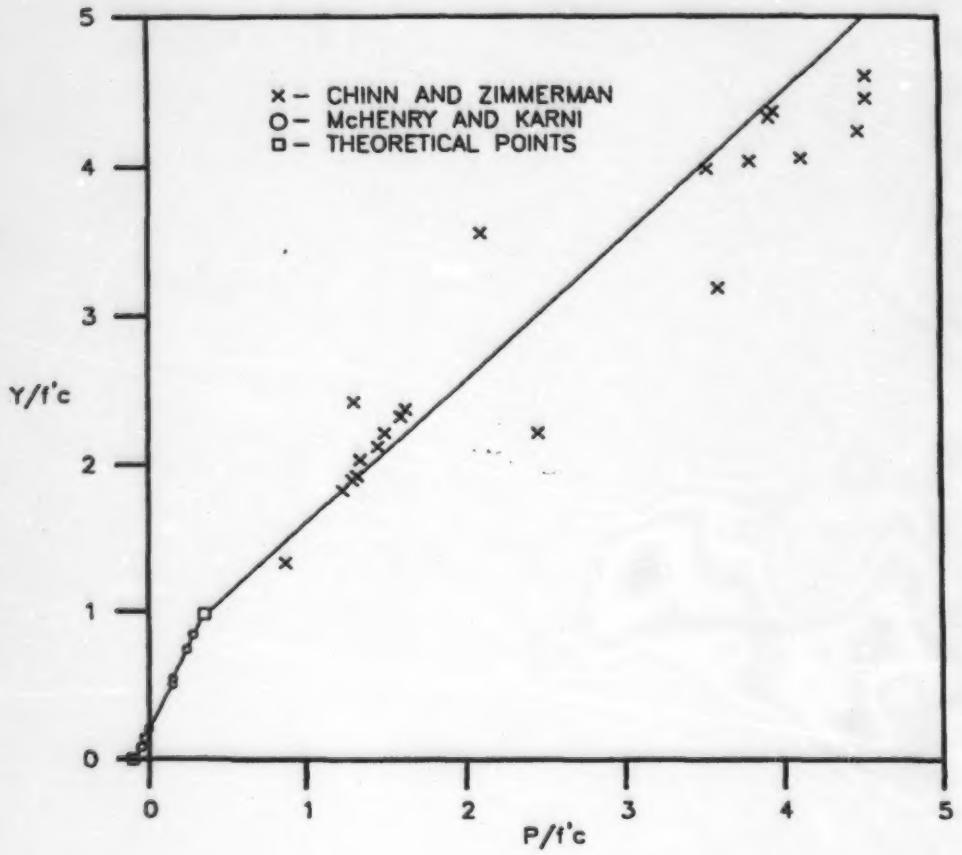

Figure I-V-4. Normalized Y Versus P at Low Pressure Levels

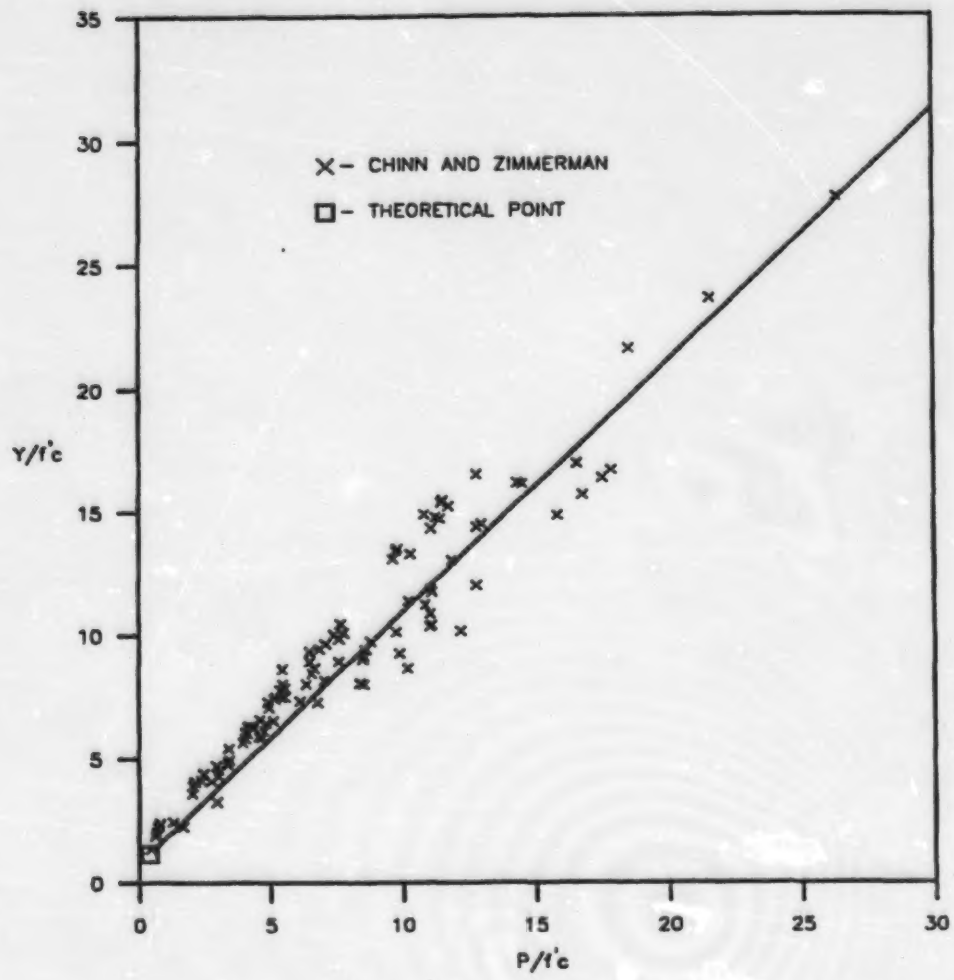

Figure I-V-5. Normalized Y Versus P at High Pressure Levels

Using the previous equations for Y, Gregson's Hugoniot stress data was reduced to provide a Hugoniot pressure vs. excess compression curve by simply subtracting 2/3 Y from stress data assuming an f_c' value of 5,000 PSI. Figure I-V-6 is a plot of this curve to 166 kilobars.

Below the precursor pressure level of 0.358 kilobars (1/3 x 0.75 kilobars), Hugoniot pressure is fit with a straight line:

$$P_H = K_O\mu$$

where K_O is determined from the empirical Young's modulus relationship (REF 13)

$$E = 33W^{1.5}(f'_c)^{0.5} \text{ PSI (W in lb/ft 3) = density}$$

and the relationship

$$K = \frac{E}{3(1-2\sigma)}$$

with a value of Poisson's ratio, σ, chosen as 0.2.

Designating the value of μ at 0.358 kilobars as μ_E, the loading Hugoniot pressure was fit with the following equations:

P_H(Kb)	RANGE OF μ
$K_O\mu$	-0 to μ_E
$0.358 + 78.62(\mu-\mu_E)$	μ_E to 0.1
$8 + 130(\mu - 0.1)$	0.1 to 0.2
$21 + 420(\mu - 0.2)$	0.2 to 0.3
$63 + 650(\mu - 0.3)$	0.3 to 0.366 (lockup point)
$73.3 + 625.5(\mu - 0.32) + 1720.19(\mu - 0.32)^2$	0.366 to 0.414
$147.3 + 39.68(\mu - 0.414)$	0.414 to 0.54 (solid - solid phase transition
$163.3 + 360.5(\mu - 0.572) + 864(\mu - 0.572)^2$	0.54 to ∞

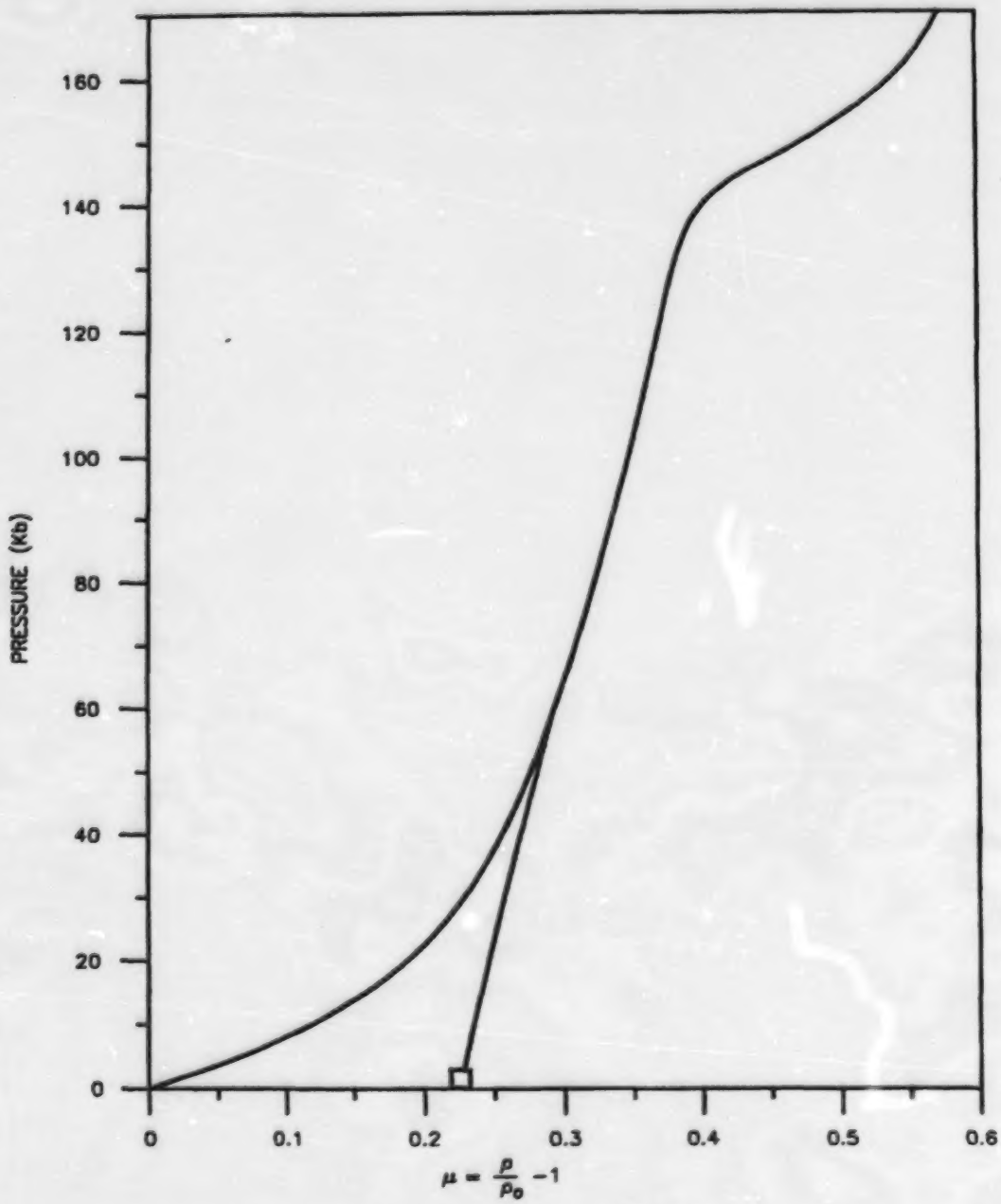

Figure I-V-6. Hugoniot Pressure

The maximum density unloading curve was fit by the equation:

$$P_H = 784\,(\mu - 0.223)$$

HULL maintains a zone variable, μ_M, for concrete which defines the maximum value of μ previously seen by the zone. For values of μ_M less than μ_E, loading, unloading and reloading all occur along the same elastic path. For values of μ_M greater than μ at lockup (0.366) loading, unloading and reloading take place along the same path. For μ_M values between these extremes, unloading and reloading up to the virgin loading curve take place along K_μ lines where K is chosen to vary linearly between K_o and 784 kilobars.

The shear modulus is calculated assuming that Hooke's law holds. That is

$$G = \frac{3\,(1-2\sigma)\,K}{2\,(1+\sigma)}$$

where K_b is the bulk modulus, $\rho\,(\frac{\partial P}{\partial \rho})$, and σ is Poisson's ratio.

Deviators are reset as though the yield surface were a von Mises surface--i.e., not a function of pressure. The resulting plastic flow rule is inconsistent with the Mohr-Coulomb yield surface proposed for the concrete. However, the alternative, such as a moving CAP model, is computationally complex and cannot be justified considering the source of the yield strength model. Strain rate effects are probably far more important than a consistent flow rule.

The Hugoniot pressure previously discussed could be combined with a Gruneisen parameter into a complete equation of state. However, Gruneisen parameter measurements for concrete are typically around 0.1. For this reason, the equation of state ignores the Gruneisen parameter and Hugoniot pressure is used as total pressure. This, of course, means that internal energy changes have no effect on pressure and the equation of state could not be used for energy deposition problems.

The variable, f_c' is set to the CGS equivalent of 5,000 PSI. If other strengths are desired, it can be changed in input. Changing f_c' will vary the fracture point and the yield surface. It will have no effect on the pressure.

Temperature is calculated for edit purposes from

$$T\,(°K) = 1.125 \times 10^{-7} I + 200$$

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